Mixture based denoising and contrast enhancement in digital radiography

I. Frosio, N. A. Borghese
AIS Lab., University of Milan

Overview

- **Statistical models and digital radiography**
- Impulsive noise removal filter
- Soft tissue filter
- Conclusion

Statistical models...

- **Principled statistical models** as an effective alternative to linear and non-linear filtering (Lucy, 1974; Richardson 1974; Shepp & Vardi, 1982);
- **Maximum likelihood / a posteriori** criterions lead to non linear cost functions;
- Filtering as a **computationally intensive iterative procedure**:Expectation Maximization (EM) (Shepp & Vardi, 1982; Geman & Geman, 1984);
- Since 90s, the necessary computational power is finally available on standard PCs.

Statistical models...

A proper statistical model for...

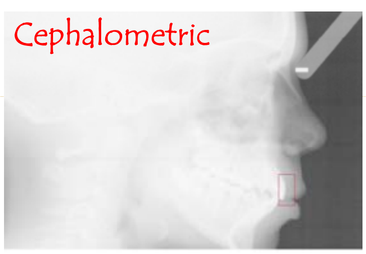
- ... Image characteristics:
  - Typical distribution of the norm of gradient:
    - Gaussian – Tikonov regularization;
    - Gibbs – TV regularization;
  - Typical-a priori grey level distribution:
    - image histogram;
    - ...

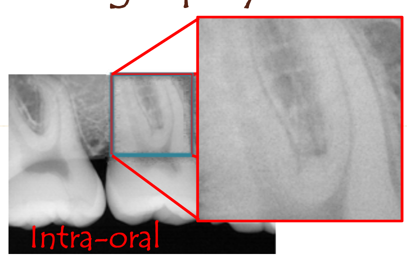
Statistical models…

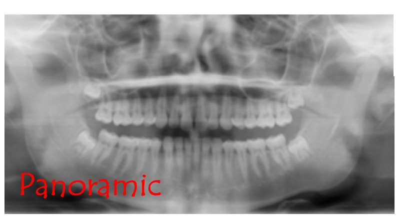
A proper statistical model for…

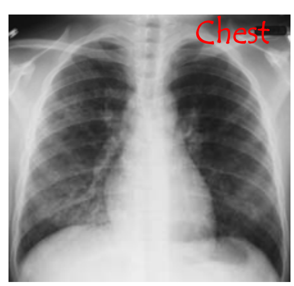
- … Image noise characteristics:
  - Distribution
    - Gaussian, Poisson, Impulsive, Speckle
    - Mixture
    - …
  - Correlation
    - White
    - PSF
    - Spatially variant
    - …

… And digital radiography









And so on…

… And digital radiography

Issues for the radiologist…

- Low contrast
- Low visibility of small anatomical details

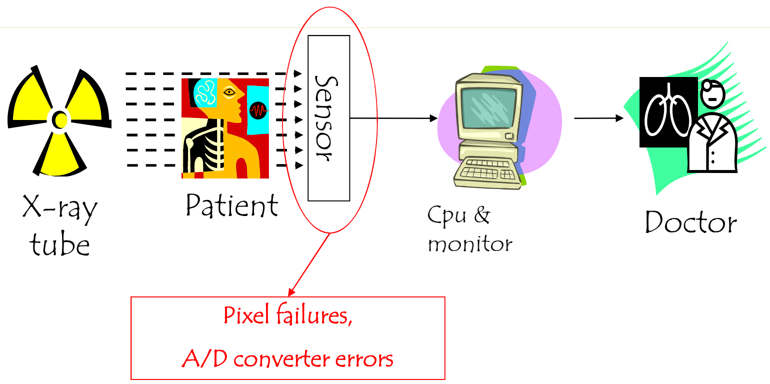
… And their (partial) solutions from the researchers:

- Contrast enhancement algorithms (e.g. γ correction)
- Feature enhancement algorithms (e.g. Unsharp Masking, UM)

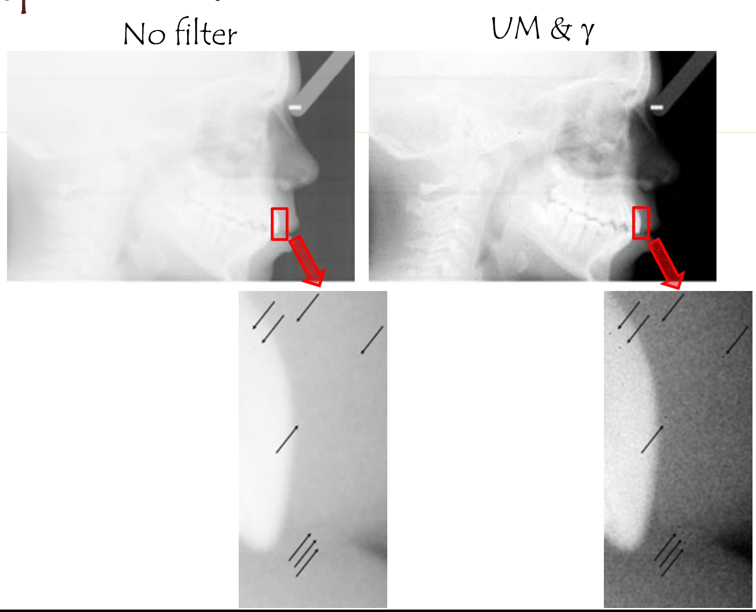
Overview

- Statistical models and digital radiography
- Impulsive noise removal filter
- Soft tissue filter
- Conclusion

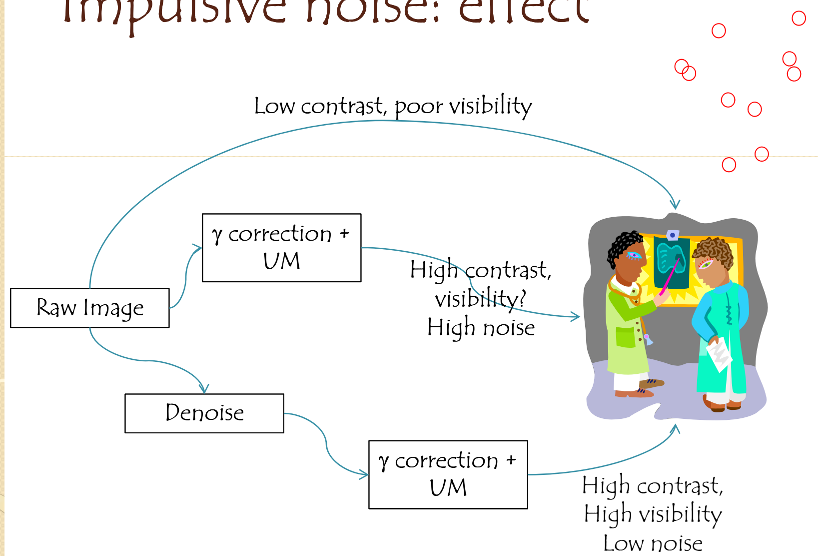
Impulsive noise: generation



Impulsive noise: effect



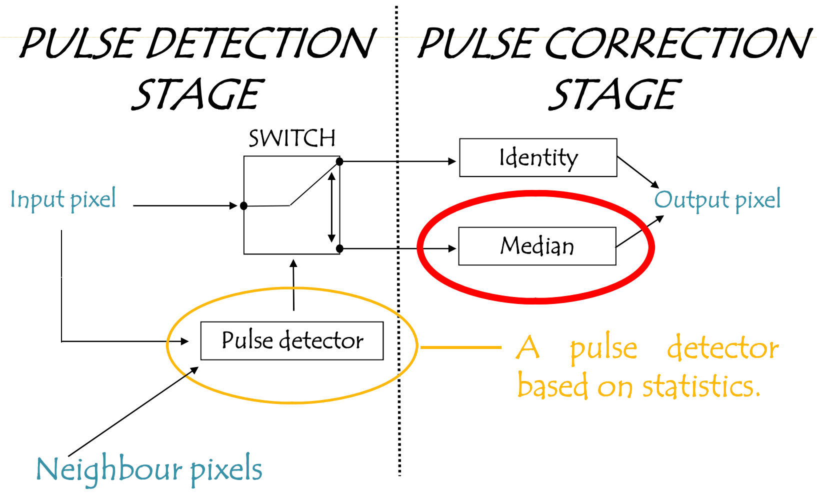
Impulsive noise: effect



Impulsive noise generates spikes

Impulsive noise: switching filter

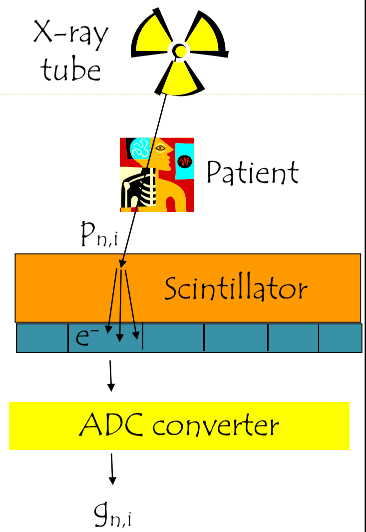
Two stages filtering



Impulsive noise: the mixture

- X-ray photons → visible photons (scintillator) → electrons (CCD sensor) → (ADC converter);
- Linear sensor:

  $g_{n,i} = G \cdot p_{n,i}$
- $p_{n,i}$: noisy number of photons for the i^{th} pixel (Poisson statistics);
- $g_{n,i}$: noisy grey level for the i^{th} pixel (??? statistics);
- G: sensor gain (from photons to grey level - unknown).



Impulsive noise: the mixture

- Changing the variable…

Mean (unnoisy) number of photons for the ith pixel.

$$p(p_{n,i} \mid p_i) = \frac{p_i^{\,p_{n,i}} \cdot e^{-p_i}}{p_{n,i}!} \quad [Poisson]$$

$$g_{n,i} = G \cdot p_{n,i}$$

$$p(g_{n,i} \mid g_i) = p(p_{n,i} \mid p_i) \cdot \left|\frac{dp_{n,i}}{dp_{n,i}}\right| = p\left(\frac{g_{n,i}}{G} \mid \frac{g_i}{G}\right) \cdot \left|\frac{1}{G}\right| = \frac{\left(\frac{g_i}{G}\right)^{\frac{g_{n,i}}{G}} \cdot e^{-\frac{g_i}{G}}}{\left(\frac{g_{n,i}}{G}\right)!} \cdot \frac{1}{G}$$

Mean (unnoisy) number of photons for the ith pixel.

Impulsive noise: the mixture

- A mixture of photon counting and impulsive noise:

$$\begin{cases} p(g_{n,i} \mid g_i) = P_{PC} \cdot p_{PC}(g_{n,i} \mid g_i) + P_{Imp} \cdot p_{Imp}(g_{n,i} \mid g_i) \\ 0 \le P_{PC} \le 1, \quad 0 \le P_{Imp} \le 1, \quad P_{PC} + P_{Imp} = 1 \end{cases}$$

- $p_{Imp}(g_{n,i}|g_i)=1/N_g$ (uniform distribution)
- N_g, number of grey levels
- P_{PC} and P_{Imp}, probabilities that a pixel is corrupted by photon counting or impulsive noise.

Unknowns

- P_{PC} and G
- Also g_i is unknown!

Supposing that the true grey level g_i is given for i=1..N, P_{PC} and G can be computed maximizing the likelihood of the data.

Impulsive noise: the mixture

- … A constrained optimization ($0<P_{PC}<1$, $0<P_{Imp}<1$, $P_{PC}+P_{Imp}=1$) should be performed.
- A simple method to constrain the solution:

$$\begin{cases} p(g_{n,i} \mid g_i) = e^{-\gamma_{PC}^2} \cdot p_{PC}(g_{n,i} \mid g_i) + \left[1 - e^{-\gamma_{PC}^2}\right] \cdot p_{Imp}(g_{n,i} \mid g_i) \\ P_{PC} = e^{-\gamma_{PC}^2} \\ P_{Imp} = 1 - P_{PC} = 1 - e^{-\gamma_{PC}^2} \end{cases}$$

- γ_{PC} and G can be computed maximizing the likelihood of the data (P_{PC}, P_{Imp} are then derived).

Impulsive noise: the likelihood

- Let us write the neg log likelihood of the measured data (grey levels of the pixels):

$$f(G,\gamma_{PC}) = -\ln[L(G,\gamma_{PC})] = -\ln\left[\prod_{i=1}^{N} p(g_{n,i} \mid g_i)\right] = -\sum_{i=1}^{N}\ln[p(g_{n,i} \mid g_i)] =$$

$$= -\sum_{i=1}^{n}\ln\left\{e^{-\gamma_{PC}^2}\cdot p_{PC}(g_{n,i} \mid g_i) + \left[1-e^{-\gamma_{PC}^2}\right]\cdot p_{Imp}(g_{n,i} \mid g_i)\right\} =$$

$$= -\sum_{i=1}^{n}\ln\left\{e^{-\gamma_{PC}^2}\cdot\frac{1}{G}\cdot\left[\left(\frac{g_i}{G}\right)^{\frac{g_{n,i}}{G}} e^{-\frac{g_i}{G}}\right] \Big/ \left[\left(\frac{g_{n,i}}{G}\right)!\right] + \left[1-e^{-\gamma_{PC}^2}\right]\cdot p_{Imp}(g_{n,i} \mid g_i)\right\}$$

Impulsive noise: the likelihood

- What about the factorial term?
- It can be approximated using the Stirling's approximation:

$$n! \approx \sqrt{2\pi n}\cdot n^n \cdot e^{-n}$$

$$\ln(n!) \approx n\ln(n) - n + \frac{1}{2}\ln(n) + \frac{1}{2}\ln(2\pi)$$

Impulsive noise: the likelihood

- With the Stirling's approximation:

$$f(G,\gamma_{PC}) \approx -\sum_{i=1}^{n} \ln\left\{ e^{-\gamma_{PC}^2} \cdot \frac{1}{G} \cdot \left[\left(\frac{g_i}{G}\right)^{\frac{g_{n,i}}{G}} e^{-\frac{g_i}{G}} \right] \Big/ \left[\sqrt{2\pi} \cdot \left(\frac{g_{n,i}}{G}\right)^{\frac{g_{n,i}}{G}+\frac{1}{2}} e^{-\frac{g_{n,i}}{G}} \right] + \left[1-e^{-\gamma_{PC}^2}\right] \cdot p_{Imp}(g_{n,i} \mid g_i) \right\} =$$

$$-\sum_{i=1}^{n} \ln\left\{ e^{-\gamma_{PC}^2} \cdot \frac{1}{\sqrt{2\pi \cdot g_{n,i}}} \cdot G^{-\frac{1}{2}} \cdot \left(g_i^{g_{n,i}} \cdot g_{n,i}^{-g_{n,i}} \cdot e^{g_{n,i}-g_i} \right)^{\frac{1}{G}} + \left[1-e^{-\gamma_{PC}^2}\right] \cdot p_{Imp}(g_{n,i} \mid g_i) \right\}$$

- For simplicity, let us define:

$$\begin{cases} H_i = 1 \big/ \sqrt{2\pi \cdot g_{n,i}} \\ Q_i = g_i^{g_{n,i}} \cdot g_{n,i}^{-g_{n,i}} \cdot e^{g_{n,i}-g_i} \end{cases}$$

Impulsive noise: the likelihood

- Some numerical problem for Q_i:
- X^x for x<127 → overflow!!!
- Better computing Q_i as follows:

$$Qi = e^{\ln(Q_i)} = \exp\{ g_{n,i} \cdot [\ln(g_i) - \ln(g_{n,i})] + g_{n,i} - g_i \}$$

$$Q_i = g_i^{g_{n,i}} \cdot g_{n,i}^{-g_{n,i}} \cdot e^{g_{n,i}-g_i}$$

$$K_i = g_{n,i} \cdot [\ln(g_i) - \ln(g_{n,i})] + g_{n,i} - g_i$$

$$Q_i = e^{K_i}$$

Impulsive noise: the likelihood

- We finally have the negative log likelihood:

$$f(G,\gamma_{PC}) \approx -\sum_{i=1}^{n} \ln\left\{ e^{-\gamma_{PC}^2} \cdot H_i \cdot G^{-\frac{1}{2}} \cdot e^{\frac{K_i}{G}} + \left[1 - e^{-\gamma_{PC}^2}\right] \cdot p_{Imp}\left(g_{n,i} \mid g_i\right) \right\}$$

- A non linear function of G and γ_{PC}
- It can be efficiently minimized through EM (a few seconds required for a 4Mpixels image @ 12bpp)

Impulsive noise: what about gi?

- The unnoisy image g_i i=1..N is unknown…
- By application of a 3x3 median filter, we obtain an image which is free from impulsive noise;
- Experimental results demonstrate that, using this image for g_i, a reliable and efficient pulse detector can be built.

Impulsive noise: pulse detector

- From γ_{PC}, P_{PC} and P_{Imp} can be computed;
- Each pixel which satisfies:

$$[P_{PC} \cdot p_{PC}(g_{n,i} \mid g_i)] < [P_{Imp} \cdot p_{Imp}(g_{n,i} \mid g_i)]$$

  is recognized as a pulse and corrected by the switching filter.
- The classification rule is chosen to minimize the classification error.