Simulation Reduction as Constraint

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Plan

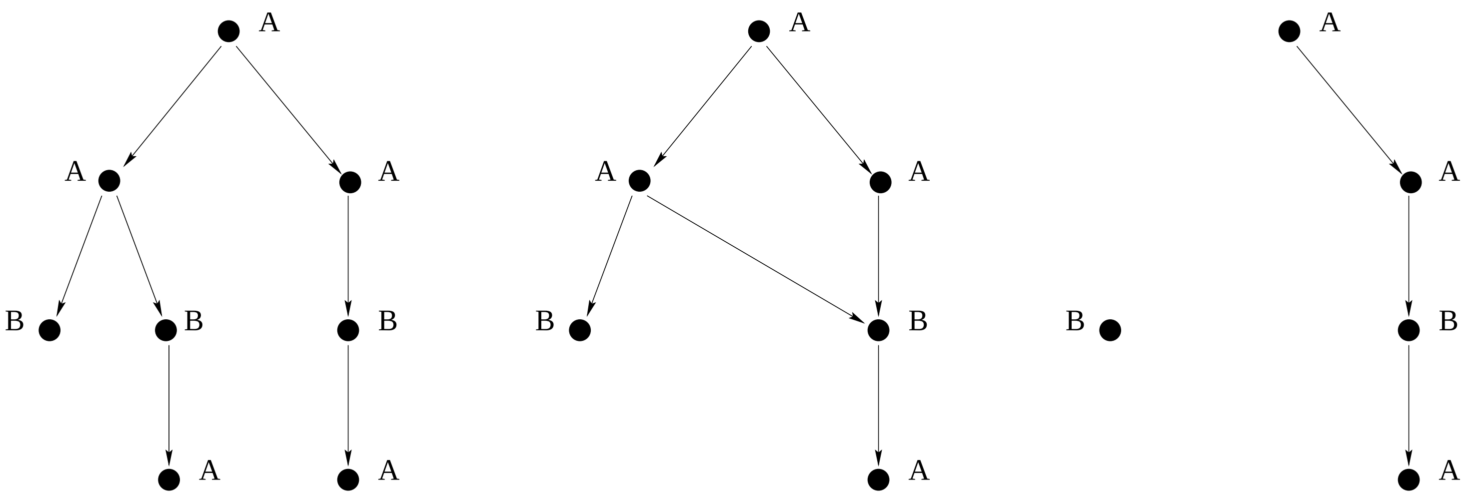
1. Bisimulation and Simulation
2. Encodings
3. Incremental Computation
4. OBDD Representation
5. Complexity Issues
6. Conclusions

What are (bi)simulations?

Equivalence relations on the nodes of a graph.

Which are their applications?

Reduce the state-spaces, compare systems, test properties, ...

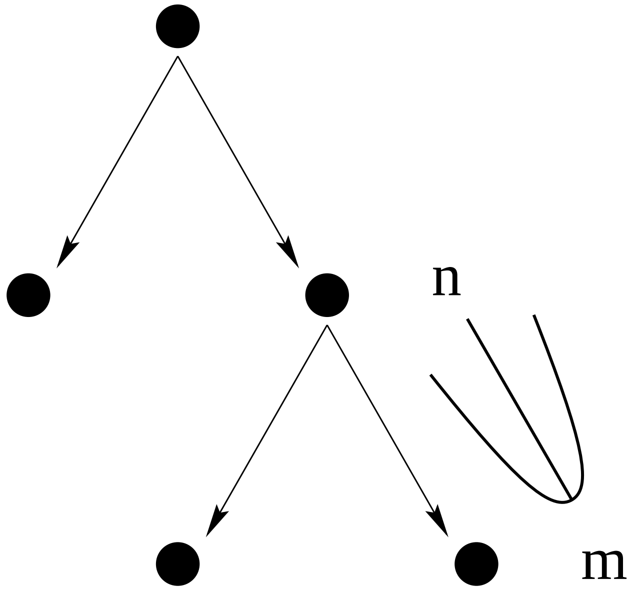


Labelled Graph Bisimulation Simulation

Colored Hypersets

$G = \langle N, E, \ell \rangle$ Colored Hyperset

$u' \in u \quad \text{iff} \quad u' \leftarrow u$



$$(E) \quad \forall u, v(u = v \leftrightarrow \ell(u) = \ell(v) \land \forall z(z \in u \leftrightarrow z \in v))$$

Bisimulation

Equality $\Leftrightarrow$ Bisimulation

$B \subseteq N \times N$ Bisimulation

$$\begin{array}{lllll}
\text{label} & uBv & \Rightarrow & \ell(u) = \ell(v) \\
\text{forw} & uBv \wedge u' \leftarrow u & \Rightarrow & \exists v'(v' \leftarrow v \wedge u'Bv') \\
\text{back} & uBv \wedge v' \leftarrow v & \Rightarrow & \exists u'(u' \leftarrow u \wedge u'Bv')
\end{array}$$

$\equiv$ Maximum Bisimulation

$$G \Rightarrow G/\equiv$$

Simulation

$S \subseteq N \times N$ Simulation

label $uSv \Rightarrow \ell(u) = \ell(v)$

forw $uSv \wedge u' \leftarrow u \Rightarrow \exists v'(v' \leftarrow v \wedge u'Sv')$

$\leq_s$ Maximum Simulation Quasi Order

$u \equiv_s v$ iff $u \leq_s v \wedge v \leq_s u$

$\equiv_s$ Maximum Simulation

$$G \Rightarrow G/\equiv_s$$

It preserves ACTL and it reduces the models more than Bisimulation

History

1971 Milner. An algebraic definition of simulation between programs

1989 Bloom. Time $O(|E|^6|N|)$.

1992 Cleaveland and Steffen. Time $O(|E|^2)$.

1993 Cleaveland, Parrow, and Steffen. Time $O(|E||N|^4)$.

1995 Bloom and Paige - Henzinger, Henzinger, and Kopke.
Time $O(|E||N|)$, Space $O(|N|^2 \log |N|)$.

2000 Bustan and Grumberg.
Time $O(|M|^2|N|^2(|M|^2 + |E|)$, Space $O(|M|^2 + |N| \log(|M|)$.

2001 Gentilini, Piazza, and Policriti.
Time $O(|M|^2|E|)$, Space $O(|M|^2 + |N| \log |M|)$.

Encodings

In the case of Bisimulation $\quad u \to u' \quad \Leftrightarrow \quad \mathit{Ack}(u) \ni \mathit{Ack}(u')$

It follows: $\quad u \equiv v \quad \Leftrightarrow \quad \mathit{Ack}(u) = \mathit{Ack}(v)$

Ackermann (1937) acyclic case.

Piazza Policriti (2001) cyclic case.

Advantages: by *rank* computation, OBDD representation.

In the case of Simulation $\quad u \leq_s v \quad \not\Leftrightarrow \quad \mathit{Ack}(u) \subseteq \mathit{Ack}(v)$

A sort of "recursive inclusion" would be necessary.

Idea: using *two* encodings

For each $u \in N$ use the codes $a(u)$ and $A(u)$.

Define the a's and the A's a way such that

$$u \leq_s v \quad \Leftrightarrow \quad a(u) \in A(v).$$

The easy ("expensive") way:

assign a different a-encoding to each node u, e.g.

$$a(u) = \{\emptyset\}^u = \underline{u}$$

determine the A-encoding using the a's.

The Constraint Defining the A's

From the Definition of Simulation

$$\begin{aligned} C(v) \quad = \quad & \{a(v)\} \subseteq \mathtt{A}_{\mathtt{v}} \subseteq \{a(u) \mid u \in N \wedge \ell(u) = \ell(v)\} \wedge \\ & \bigwedge_{\ell(u)=\ell(v)} (a(u) \in \mathtt{A}_{\mathtt{v}} \leftrightarrow \{a(u') \mid u \to u'\} \subseteq \bigcup_{v \to v'} \mathtt{A}'_{\mathtt{v}}) \end{aligned}$$

$$Sys_G \quad = \quad \bigwedge_{u \in N} C(u)$$

Max Simulation as Max Solution

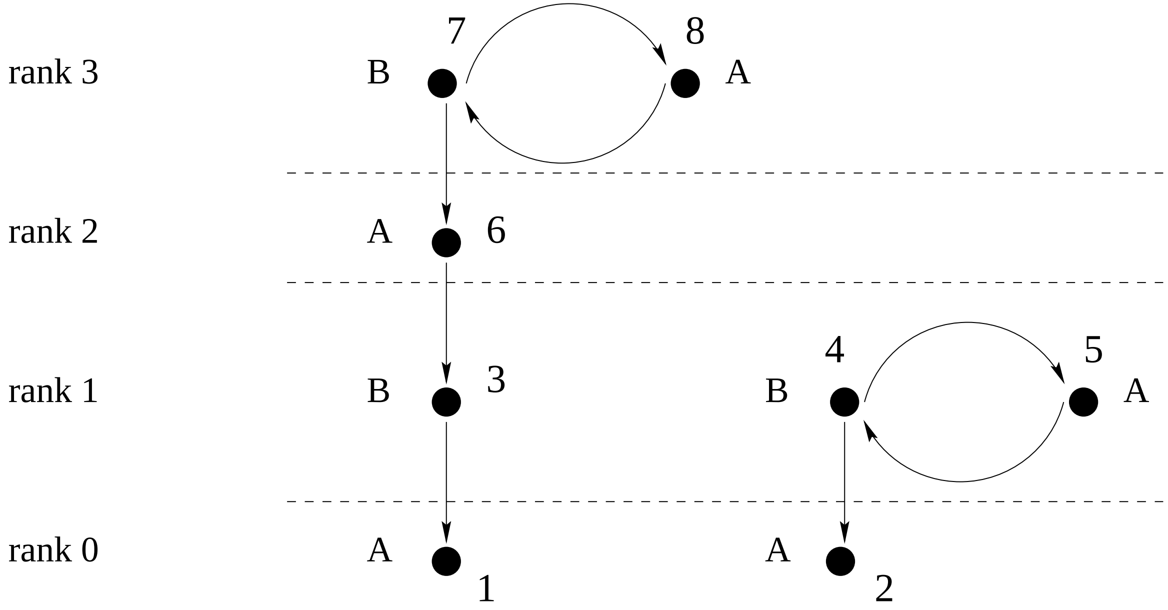
A solution Sol_1 of Sys_G $\quad\Rightarrow\quad$ A simulation $\leq_{Sol_1}$ on G

$$u \leq_{Sol_1} v \qquad \text{iff} \qquad a(u) \in A_1(v).$$

Let $Sol_1 \sqsubseteq Sol_2$ iff $\forall u \in N(A_1(u) \subseteq A_2(u))$

$$SM \text{ the } \sqsubseteq\text{-max sol. of } Sys_G \Rightarrow \begin{cases} a(u) \in A_{SM}(v) & \Leftrightarrow \quad u \leq_s v \\ \\ A_{SM}(u) = A_{SM}(v) & \Leftrightarrow \quad u \equiv_s v \end{cases}$$

The notion of *rank*



$rank(u)$ = length of the longest path from u to a leaf in the scc-graph.

Incremental Computation using *rank* - Basic Ideas (I)

Unfortunately

$$u \leq_s v \quad \not\Rightarrow \quad \mathit{rank}(u) \leq \mathit{rank}(v)$$

$$u \equiv_s v \quad \not\Rightarrow \quad \mathit{rank}(u) = \mathit{rank}(v)$$

At step i consider only the nodes up to *rank* $i-1$.

If at the end of step i $A(u) = A(v)$, then re-assign $a(u) := a(v)$.

At step $i+1$:

- compute the A-encoding of *rank* i;
- upgrade the old A-encoding with the nodes at *rank* i.

Incremental Computation using *rank* - Basic Ideas (II)

Let $rank(u), rank(v) < i$.

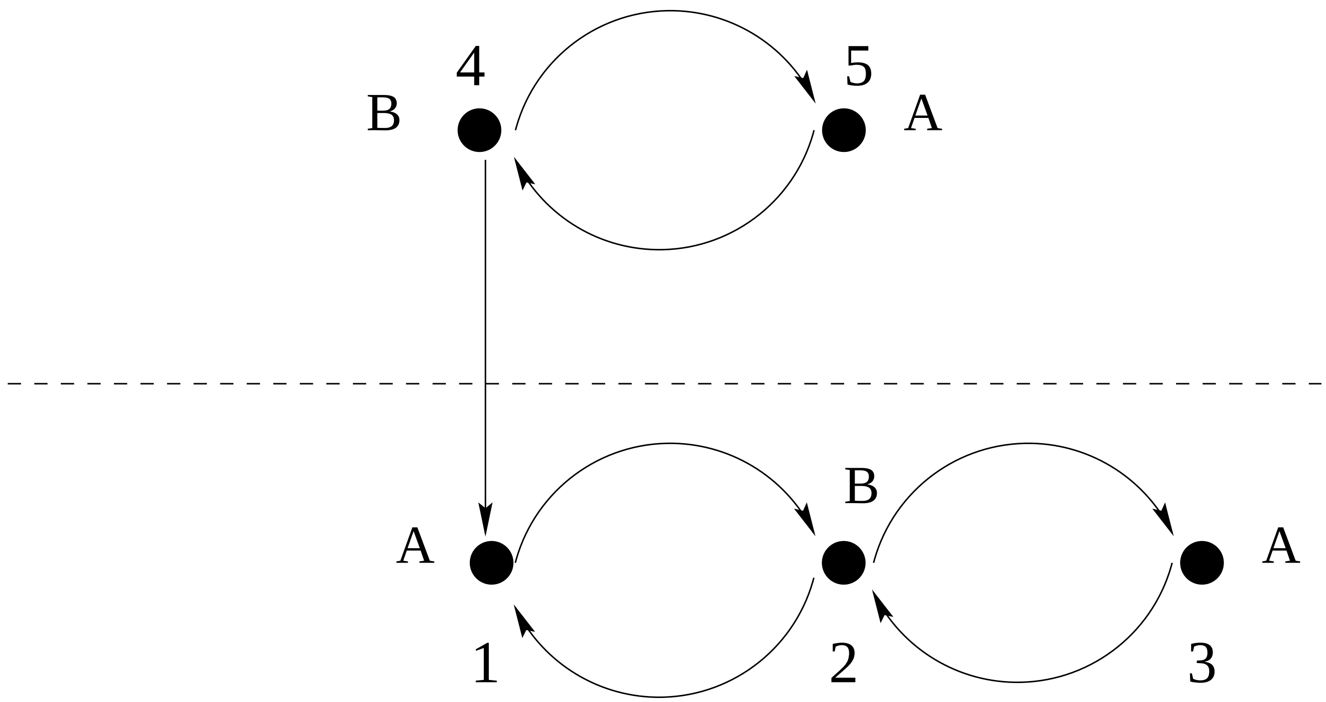
It is never necessary to remove $a(u)$ from $A(v)$.

If at step i $a(u) \notin A(v)$, this is definitely true and $u \not\leq_s v$.

If at step i $A(u) = A(v)$, this is definitely true and $u \equiv_s v$.

If at step i $A(u) \neq A(v)$, this is definitely true and $u \not\equiv_s v$.

Example



At the end of step 1 $A(1) = A(3) = \{1, 3\}, A(2) = \{2\}$,
hence $a(3) = a(1) = 1$.

At the end of step 1 $A(1) = A(5) = \{1, 5\}, A(2) = A(4) = \{2, 4\}$,
hence $a(5) = a(1) = 1, a(4) = a(2) = 2$.

Incremental Computation using *rank* (I)

At step i using the nodes at ranks $0, 1, 2, \ldots, i-1$
we have the partial solution

$$a(1) = \underline{1} \quad \ldots \quad a(k) = \underline{k'}$$

$$A(1) = \{a(1), \ldots\} \quad \ldots \quad A(k) = \{a(k), \ldots, \},$$

$$\text{with } a(h) = a(j) \Leftarrow A(h) = A(j)$$

$$\text{and } max\{a(1), \ldots, a(k)\} = \underline{m},$$

Incremental Computation using *rank* (II)

At step $i+1$ we have to:

consider all the nodes at rank $0, 1, 2, \ldots, i$;

assign to the s nodes at rank i an a-encoding

$$a(k+1) = \underline{m+1} \quad \ldots \quad a(k+s) = \underline{m+s};$$

if h is a node at rank $0, 1, 2, \ldots, i-1$, consider

$$A(h) \subseteq \mathtt{A_h} \subseteq A(h) \cup \{a(j) \mid rank(j) = i+1 \wedge \ell(j) = \ell(h)\} \wedge$$

$$\bigwedge_{rank(j)=i+1 \wedge \ell(j)=\ell(h)} (a(j) \in \mathtt{A_h} \leftrightarrow \{a(t) \mid j \to t\} \subseteq \bigcup_{h \to v} \mathtt{A_v});$$

Incremental Computation using *rank* (III)

if h is a node at rank i, consider

$$\{a(h)\} \subseteq \mathtt{A_h} \subseteq \{a(j) \mid rank(j) \leq i+1 \wedge \ell(j) = \ell(h)\} \wedge$$

$$\bigwedge_{rank(j) \leq i+1 \wedge \ell(j) = \ell(h)} (a(j) \in \mathtt{A_h} \leftrightarrow \{a(t) \mid j \to t\} \subseteq \bigcup_{h \to v} \mathtt{A_v});$$

if $A(1) = \{a(1), \dots\} \quad \dots \quad A(k+s) = \{a(k+s), \dots\}$ is the $\sqsubseteq$-max. solution and $A(h) = A(j) \wedge a(h) < a(j)$, then re-assign to j the a-encoding $a(h)$.

Fully Incremental OBDD representation

At step i if $rank(u) = i - 1$ consider the OBDD

$$E(u) \quad = \quad \bigvee_{u' \in Big(u)} a(u')$$

where

$$Big(u) \quad = \quad \{u' \mid u \to u' \wedge a(u') \notin \bigcup_{u \to u'' \wedge u'' \neq u'} A(u'')\}.$$

It is never necessary to upgrade $E(u)$!

If at step $j \geq i$ $A(u) = A(v)$, do not compute $E(v)$!

The collection of the OBDD's is a symbolic representation of $G/\equiv_s$.

Complexity Issues

Exploit existing constraint solvers, such as $CLP(\mathcal{SET})$.

Trade-off between the expressive power and the time complexity.

Easy to implement a constraint solver running in $O(|N|^3|E|)$.

Conclusions

We propose an alternative way to:

- efficiently compute $G/\equiv_s$
- produce simultaneously $\equiv_s$ and an OBDD representation

Under investigation:

- algorithmic complexity aspects and testing
- operations on the OBDD representation